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The effect of small-group instruction on the problem-solving achievement of children

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THE EFFECT OF SMALL-GROUP INSTRUCTION ON THE PROBLEM-SOLVING ACHIEVEMENT OF CHILDREN

MASTER'S PROJECT

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CHAPTER I

INTRODUCTION

Children are often expected to find solutions in isolation in an educational setting. Children could benefit from sharing solution strategies in small-group experiences.

Statement of the Problem

Children need additional experiences with problem solving in mathematics. One means of providing these experiences is to increase the amount of word problems in the mathematics curriculum. Children that are given the option of small-group activity in the solution of word problems may demonstrate higher achievement than those students that must find solutions individually.

Purpose of the Project

The East Liverpool City School District has an elementary mathematics program based primarily on computational math. Through this project I hope to show the benefits of an increase in problem solving experiences for students.

Scope of the Project

The project will take place at La Croft Elementary in the East Liverpool City School District.

The testing part of my project will involve two elementary grade classrooms. The experimental group will be given word problems in a small-group setting. The control group will be given the usual instruction in word problems from the Houghton-Mifflin text.

Definitions

algorithm- A mathematical sentence. (5+2=7)

cognitive processing capacity- Mental functions can be characterized by the way information is stored, accessed, and operated on. (Romberg & Collis, 1985)

curriculum- The content of the courses offered in the schools.

mental functions- An intake register through which information from the environment enters the system. (Romberg & Collis, 1985)

M-space- A working or short-term memory in which the processing occurs. (Romberg & Collis, 1985)

problem solving- The inquiry for solutions to open-ended, non-routine situations that require observing, gathering data, making predictions, testing the predictions, arriving at tentative solutions, and testing the solutions. Or quantitative situations presented in written form in which a question or questions are asked without an accompanying statement concerning the mathematical operation required. (Riedesel, 1985)

strategies-

a. direct modeling
The child uses a one-to-one ratio with fingers or physical objects.

b. counting-on-from-first
The child begins counting foward with the first addend in the problem.
In (3+4=?) the child would count 3[pause] 4,5,6,7. The answer is 7.

c. counting-on-from-larger
Identical to counting-on-from-first, except that the child begins counting foward with the larger of the two addends. In (3+4=?) the child would count, 4 [pause] 5,6,7. The answer is 7.

d. recall
The child has retained the math fact.
(Carpenter & Moser, 1984)

verbal problems- Quantitative situations presented in oral form in which a question or questions are asked without an accompanying statement concerning the mathematical operation required. (Riedesel, 1985)

word problems or story problems — Quantitative situations presented in written form in which a question or questions are asked without an accompanying statement concerning the mathematical operation required. (Riedesel, 1985)

General Hypothesis

Students that experience small group solutions of word problems may show higher achievement levels in word problem solution.

CHAPTER II

Literature Review

Problem solving is the central focus of mathematics in the 1980's, according to An Agenda for Action, published by the National Council of Teachers of Mathematics. Problem solving in mathematics demands a synthesis of abilities to produce solutions.

From the wide range of problem solving activities available I am concentrating on word problems. Word problems provide a unique combination of two elements in problem solving. The word problem contains one or more numerical algorithms in written form with an unknown solution procedure. This provides a measurable outcome for analysis of a student's ability to think and reason.

First, I will investigate the number of word problems in current math texts. Next I will discuss the lack of performance in verbal problems. Then I will review the strategies students use in the solution of word problems. Then I will consider what knowledge is necessary to be successful in the solution of word problems. Last, I will discuss some necessary elements of a problem solving environment.

Mathematical Texts

In contrast to the mandated emphasis on problem solving in the 1980's, mathematical texts are illustrating a lack of opportunity with problem solving experiences. To measure this variable, (Mc Ginty, Van Beynen, Zalewski, 1986) tabulated the number of story problems in current texts, in comparison to textbooks in the past. Story problems were chosen not because story problems are the only means of obtaining problem solving experiences, but because they are the "most predominate form of problem solving found in both past and current texts. " Current texts have less than 10% "enrichment activities," "brain teasers," or "think problems." (p. 593)

His findings show a dramatic decrease in the amount of story problems incorporated into current texts.

A Comparison of Textbooks

Year	Written Words	Word Problems	Drill	Pages
1924	69,000	1,510	3,700	400
1944	47,000	1,620	4,400	310
1984	34,000	510	5,800	390

Numbers are approximate.

(Mc Ginty, Van Beynen, Zalewski, 1986, p.595)

This study demonstrates a need for additional experiences with story problems. Also, note the comparison in the number of drill problems. It is excellent evidence that computational problems have increased in texts, while the number of story problems have declined.

It would be futile to place blame on the publishers of textbooks for this occurrence. They are only responding to the needs of educators. It is possible that the avoidance of word problems in mathematical texts is linked to the lack of performance that students experience in the solution of story problems that cannot be solved with a familiar algorithm. In (Carpenter, Lindquist, Brown, Kouba, Silver, Swafford, 1988) a national assessment of mathematical trends since 1973 cites the "greatest cause for concern" is students performance in word problems. (p. 41)

It would be logical to assume that the difficulty students experience would make demands on textbook publishers to increase the amount of story problems. Instead, there is an increase in repetition of computational problems. Perhaps it is easier to emphasize an area in which students experience higher achievement.

Problem-Solving Research

(Carpenter, Corbitt, kepner, Lindquist, Reys, 1980) researched lack of performance in verbal problems. Their original position states that students do not experience the same amount of difficulty with all types of verbal problems. The difficulty depends on the semantic structure and complexity of the problem.

Carpenter disagrees with instruction that places emphasis on choosing one correct operation, stating that the result is damaging problem solving skills. He also criticizes using a "key word" technique. He believes children will rely on this technique too often. The result may be high performance on simple one-step problems, and low performance on other types of verbal problems.

Story problems that cannot be solved with a familiar algorithm cause a high degree of difficulty for students. His findings demonstrated a 90% performance on a verbal whole-number addition story problem, only requiring one simple calculation. The performance was only 8% points lower when students were required to read the problem. This contradicts a former theory that reading skills alone are the cause of low scores on story problems. The same group of nine-year-olds demonstrated a 47% performance on a verbal problem that could not be solved with one simple operation.

Emphasis on simple one-step problems does little for the improvement of problem solving skills. Instead of searching for solutions, students in this study performed a single operation to the numbers given in the problem. In problems with extraneous data, many students incorporated all the numbers given into their calculation. The result is a lack of logical analysis and instead illustrates student preoccupation with <u>any</u> solution. (No matter how meaningless.)

(Carpenter, Corbitt, Kepner, Lindquist, Reys, 1980) described texts that negate problem solving skills.

> A unit on multiplication frequently includes a lesson exclusively on multiplication word problems. Such a lesson may expose children to a number of different types of problems that can be solved by multiplication. But once students realize that all problems can be solved by multiplication, many of them stop analyzing the problems with any care and may even stop reading them. (pg. 11,12)

This lack of analysis of story problems may be only a small part of students' difficulty in formulating solutions. The next section of my review deals with the strategies students exhibit in finding solutions for addition and subtraction word problems.

(Cummins, 1988) describes the Piagetian view that Carpenter and others have used as an premise to research the solution performance characteristics of children. It is based on the concept that problems are troublesome for children "because the capacities required to process the problem are not yet possessed by the child."(p.406)

One model proposed by (Carpenter & Moser, 1984) in a three year longitudinal study identified the stages children pass through over a period of time in the solution of addition and subtraction word problems. Eighty eight children were interviewed eight times on an individual basis. Each time children were asked to solve two addition and four subtraction problems. The word problems were given in order of increasing difficulty. They were typical of the word problems given in elementary texts.

Their findings concluded that the strategies differed for addition and subtraction. Three basic levels of addition strategies were identified as: direct modeling, counting strategies, and recall of numbers. More intricate strategies were used with subtraction than with addition.

Other conclusions of the study: Children do not always choose the same strategies. When children have several strategies available, they use them interchangeably. Children will sometimes revert to a less-efficient strategy even after they have acquired a more sophisticated method for solution. Some children would fall back on direct modeling when physical objects were made available.

This study provides accurate information on children's solution strategies and how they change over time. It does not resolve the question of what "knowledge structures are necessary to represent problems." (p. 199) The study used data based on the final sample after three years. As a control for instruction, schools using the same text were selected.

The primary implication for instruction suggests that instructors of the early grades are not capitalizing on the informal mathematical skills that children bring to school. Instruction should use the internalized problem solving ability of children.

Current mathematical instruction leaps from the concrete to the memorization of math facts. At the end of the study, in the middle of the third grade "11% of the children had not demonstrated a mastery of facts less than 10, and 30% had not demonstrated a mastery of facts greater than 10."(p. 196) The children were classified as fact users if they used number facts on two thirds of the problems. The implications of this information suggests that children do not retain math facts as quickly as teachers may assume. Fifty-eight percent of the children are using counting strategies in the beginning of the third grade, when many teachers chastise students for "still using your fingers!"

The following study will consider what knowledge is necessary for students to be successful in the solution of word problems.

(Romberg and Collis, 1985) identified two groups of children who differed in their mental functions. The capacity of "memory ... to process information appears as a fundamental characteristic of cognitive development in a number of theories." (pg. 375) Children have limitations in their ability to deal with complex tasks. That capacity is defined as "M-space." (pg. 376) (Please see definitions.)

To identify the groups, they tested 139 children in an Infant School in Hobart, Australia. They used five tests to determine M-space of the children. They also used ten tests of cognitive-processing capacity. The testing identified "six empirically well-defined sets of children with specific cognitive characteristics. Factor analysis and cluster analysis were used to interpret the data." (pg. 377) They contrasted "four children in the lowest group (Group-L) with seven children in the highest group (Group-H)." (pg. 377) They all were children entering the third grade in the fall.

They tabulated frequency of correct responses and general strategies of the two groups. The findings confirm informal observations of classroom experience. There is a discrepancy in achievement between the "haves" and the "have-nots."

In the first set of verbal problems requiring no regrouping Group-L scored 58% and Group-H scored 87%. In problems requiring regrouping Group-L scored 42% and Group-H scored 80%.

Group-H children tended to use direct modeling or counting at the beginning of the study. At the end of the study, only 7% of the children used direct modeling, and 32% still relied on counting strategies. Many of the children began to use the sentence algorithm strategy. They also were computing the less difficult problems "in their heads." (pg. 379) Group-L children used an incorrect algorithm most often. These children rarely used counting strategies and used direct modeling on the less difficult problems.

The authors make no claim of generalizability or causality. They do believe their findings in "describing relationships between problem-solving success and the use of strategies" will hold in that: (a) "Children differ in cognitive capacity to function with quantitative verbal problems, and (b) Children who differ in their cognitive-processing capacity also differ in the strategies they use to solve the same verbal problems and differ in their success in finding correct answers." (pg. 380)

The study focuses on two groups of children that need to be educated. The children "who have the capacity to reason about quantitative problems ... are confident that the procedures they use are satisfactory." Their is a second group "whose capacity to reason about quantitative problems is suspect... and have not acquired other skills like direct modeling or counting strategies that would help them solve verbal problems." (pg. 381)

As an educator, it is far too often an objective of my profession to segregate the "haves" from the "have-nots." Far too often the "have-nots" are frustrated by the educational system. Could the process of these students working together in small-groups for solutions to story problems improve the achievement of the group? It is my prediction that it will. I am hoping that students will model solution strategies that they discover are successful in the solution of word problems. I predict this treatment will utilize student's informal mathematical knowledge.

Problem-Solving Environment

Problem-solving cannot take place without an agreeable environment. One important aspect of this environment is the teacher's acceptance of errors as a necessary component of the problem-solving process. This must be demonstrated in classroom practice to be meaningful.

Children will only persist in finding solutions if the instructor demonstrates the acceptability of spending all the time allotted in the solution of one problem. "Children must view mathematical problems as personal challenges." (Sowder and Sowder, 1988, p. 46)

Success is achieved by "the ways in which they (teachers) communicate their expectations to the children and thus attempt to place the children under certain obligations for their conduct in the classroom." (p. 46) The children must be the ones responsible for their learning and conduct in the classroom. Disagreements among partners must be solved by the parties involved. "By giving the children the primary responsibility for their learning and conduct in the classroom, the teacher was generally successful in helping them develop productive working relationships." (p. 47)

The teacher has an obligation to the children of "viewing children's solution attempts as expressions of their mathematical thinking that should be treated with respect." (p. 47) This is a critical part of the process. Any attempt by the teacher to manipulate the reasoning of the children resulted in children's withdrawal of solution explanations.

In conclusion, teachers that need a problem-solving environment must interact with their students to develop successful strategies for dealing with problems that will occur in small-group instruction. This interaction is necessary in the "development of the wisdom and judgment that characterizes the expertise of successful teachers." (p. 47)

CHAPTER III

DESIGN

Type of Design

The design used in this project was an Experimental Design. Both groups were given a pretest and posttest. The experimental group was given the treatment.

Two third grade classes were participants in the project. One class was the experimental group and the other class was the control group. At the end of the project gain scores of pretest and posttest are compared.

Participants

The participants of the project were two heterogeneous third grade classes at La Croft Elementary in the East Liverpool City Schools. Forty students participated in the project. One third grade class was used as the control group and the other third grade class was used as the experimental group. The range of the ability of the students is as close to equal as possible.

Apparatus

The test used five basic types of addition and subtraction problems. These problems were selected because they were representative of those commonly used in elementary texts. The problems were identically formatted to those used in previous research on children's problem-solving ability. The original problem format can be found in the Development of Children's Problem Solving Ability in Mathematics by Mary S. Riley from the University of

Pittsburgh.(p. 160)

The test consisted of ten word problems. This is the maximum number considered by myself and two other third grade teachers to be manageable by third grade children. Our math text does not exceed this number of word problems for any one evaluation or practice exercise.

The test scores of the pretest to posttest were compared using a *t* test to determine if the experimental group experienced significant gains.

The control group was given word problems from the Houghton-Mifflin text. The students worked individually for solutions using manipulatives and drawing materials. Solutions were shared by the class.

The experimental group also began the project using problems from the text, but evolved into using situational word problems. Students worked in small-groups also using drawing materials and manipulatives as aids. One example of this type of problem evolved from plans to make puppets in the class for a puppet show. The students tabulated the number of puppets that needed to be made, and the number of students that needed to be involved as puppeteers. This led to a discussion of the cost of the materials.

Procedure

First I met with my principal and received permission to carry out my project. (Appendex) He reviewed the letter I was to send home to parents. (Appendex)

Then I sent a letter home to parents explaining the project to parents. The letters were sent home with the children, signed by the parents and returned to school.

After I received permission from my principal and the parents of my students, I was ready to begin my project.

Next I designed a test to be used as both a pretest and posttest. Three mathematics teachers reviewed my test, and revisions were made.

Then I planned what problem-solving exercises I would include in my lesson plans for the six-week period. My weekly lesson plans consisted of two fourty-five minute sessions utilizing word problems from the Houghton-Mifflin text appropriate for the computational abilities of my students.

My next step was to devise a procedure to choose an experimental group and a control group. I accomplished this by chosing two room numbers at random. My first choice was my control group and my second choice was the experimental group.

Both groups were then given the pretest. Instruction content and materials were identical for both groups. The control group solved problems on a one-by-one basis. The lesson plan for the control group included a word problem, then solution time by students, followed by a sharing of solution procedure.

The experimental group had five four-member team groups. Each group was given a word problem for solution. The group had extended time for solution. At the end of the time period the groups shared solutions.

The last step of my project was to give both groups the posttest. The results of the pretest and posttest were examined and the gain scores were compared. A *t* test was used to see if the experimental group gain scores were significantly higher than the control group gain scores.

Operationally Defined Hypothesis

The children in a small group situation for problem solving will have significantly higher pretest-posttest gain scores than the individual problem-solving method.

CHAPTER IV

ANALYSIS OF DATA

A *t* test was run and no significant difference was found in results of the pretest- posttest gain scores between the group receiving small-group instruction and the group receiving traditional instruction.

CONTROL GROUP

STUDENT	PRETEST	POSTTEST	GAIN
1.	80	80	+0
2.	90	100	+10
3.	90	90	+0
4.	80	90	+10
5.	70	80	+10
6.	80	80	+0
7.	80	80	+0
8.	70	80	+10
9.	70	80	+10
10.	50	50	+0
11.	40	50	+10
12.	40	60	+20
13.	0	30	+30
14.	50	70	+20
15.	80	80	+0
16.	50	60	+10
17.	60	80	+20
18.	80	90	+10
19.	20	30	+10
20.	80	90	+10

EXPERIMENTAL GROUP

STUDENT	PRETEST	POSTTEST	GAIN
1.	50	50	+0
2.	80	90	+10
3.	70	90	+20
4.	70	70	+0
5.	70	70	+0
6.	60	60	+0
7.	70	90	+30
8.	80	90	+10
9.	60	80	+20
10.	70	90	+20
11.	80	80	+0
12.	90	90	+0
13.	50	50	+0
14.	40	50	+10
15.	80	90	+10
16.	60	80	+20
17.	70	90	+20
18.	80	80	+0
19.	60	70	+10
20.	70	90	+20

CHAPTER V

DISCUSSION

Conclusions about the hypothesis

The children in small group situations for problem solving <u>did</u> <u>not</u> have significantly higher pretest-posttest gain scores than those children working individually for solutions.

Interpretation

Both experimental and control groups had significantly higher pretest-posttest gain scores. Therefore, the treatment of students working in small groups for solutions <u>was</u> <u>not</u> the critical factor.

The critical factor for student achievement in this study is the instruction and the amount of instructional time I devoted to word problems.

Recommendations

The East Liverpool City Schools have currently recognized the need to improve the problem-solving achievement of our students. My recommendation would be to form a Mathematics Curriculum Committee to study available resources to improve the problem-solving abilities of our students. The committee should include members of the administration, teachers, board and interested community members.

First the committee should consider results of current research on problem-solving before any changes are made in the curriculum.

Next the committee needs to change the curriculum to include an increase in the amount of problem-solving experiences for students including a testing procedure to study results.

Then professional training must be made available for a core group of mathematics teachers on all levels to study improved techniques and materials for problem-solving. This core group then must be given opportunities to share this information with the staff of each building.

Only after the training period has taken place should the change in curriculum be implemented. Without teacher preparation any changes made in curriculum would be futile. Teachers unconvinced of a need for change or the methods to implement the change would be ineffective.

Limitations

My project was limited to one grade level and one school. That was because my access to students is limited as a classroom teacher.

LaCroft Elementary School

2460 Boring Lane

EAST LIVERPOOL, OHIO

October 9, 1989

Dear Parents,

I am presently working on my Masters Project at the University of Dayton. The project deals with the teaching of mathematics.

The project will involve the Third Grade classes at La Croft Elementary. The children will be given a pretest, then given instruction in math and given a posttest.

I would appreciate your cooperation in this project. Please sign below and return.

Sincerely,

Catherine Horrocks

I give my permission for my child to participate in the project.

____
Parent's Signature

____
Date

Lancers

La Croft Elementary School

2460 Boring Lane

EAST LIVERPOOL, OHIO 43920

09 October 1989

Dear Sir,

I give my permission and my encouragement to Catherine Horrocks to complete her Masters' Project for the University of Dayton. I understand that she will be working on problem-solving abilities with third grade students.

Sincerely,

Richard E. Wolfe

Richard E. Wolfe, Principal

NAME ______________________________

1.STEVE HAS 15 DOMINOES. HE HAS 8 MORE DOMINOES THAN BOB. HOW MANY DOMINOES DOES BOB HAVE? ______________

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2. TRACY HAS 14 NINTENDO TAPES. SHE HAS 4 TAPES LESS THAN CINDY. HOW MANY NINTENDO TAPES DOES CINDY HAVE?

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3. JOE AND TERRY HAVE 16 MARBLES ALTOGETHER. JOE HAS 9 MARBLES. HOW MANY MARBLES DOES TERRY HAVE?

____

---

4. BOB HAD SOME TOY CARS. THAN TOM GAVE HIM 8 MORE TOY CARS. NOW BOB HAS 15 CARS. HOW MANY DID BOB HAVE IN THE BEGINNING?

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5. BUTCH HAD SOME PUZZLES. THEN HE GAVE 5 PUZZLES TO BILL. NOW BUTCH HAS 14 PUZZLES. HOW MANY PUZZLES DID BUTCH HAVE IN THE BEGINNING?

____

6. SUE HAS 16 GAMES. SHE HAS 9 MORE GAMES THAN CAROL. HOW MANY GAMES DOES CAROL HAVE?

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7. DONNA HAS 13 PENCILS. SHE HAS 7 PENCILS LESS THAN TOM. HOW MANY PENCILS DOES TOM HAVE?

____

---

8. BILL AND RICH HAVE 18 MODEL AIRPLANES ALTOGETHER. RICH HAS 8 MODEL AIRPLANES. HOW MANY MODEL AIRPLANES DOES BILL HAVE?

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9. BETH HAS SOME BOOKS. THEN TERRY GAVE HER 9 MORE BOOKS. NOW BETH HAS 17 BOOKS. HOW MANY BOOKS DID BETH HAVE IN THE BEGINNING?

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10. SALLY HAD SOME DOLLS. THEN SHE GAVE 6 DOLLS TO SUSAN. NOW SALLY HAS 12 DOLLS. HOW MANY DOLLS DID SALLY HAVE IN THE BEGINNING?

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