Winter 2004

2004 (Winter)

University of Dayton. Department of Mathematics

Abstracts of the Colloquium Talks: Winter 2004
Department of Mathematics

Mathematical Aspects of Financial Portfolio Optimization (Jan. 15)
Prof. Srdjan Stojanovic
Abstract: We shall review classical results in continuous time financial portfolio optimization due to R. C. Merton (1997 economics Nobel Prize winner), for assets with Log-Normal price-dynamics. Then we shall discuss some recent results of the speaker, on extensions of Merton's theory, such as the theory for portfolios of assets with general non-linear, possibly degenerate, Markovian price-dynamics. Such portfolios may include options as well. Numerical simulations will be shown, some of them live.

B**oundary Value Problems for Dynamic Equations on Time Scales** (Jan. 22)
Dr Christopher C. Tisdell
Abstract: Boundary value problems (BVPs) are mathematical equations that are useful for modelling the world in which we live. With the development of the new 'time scale calculus', BVPs on time scales can further extend our modelling capabilities to describe those processes that may change smoothly at one level, but irregularly at another level.

In the talk I will give an introduction to the time scales calculus and then discuss the existence of solutions to BVPs on time scales. Much of the talk will be at a very general level.

Optimization of Utility of Consumption and Terminal Wealth (Feb. 3)
Andrew Ellett
Abstract: In this talk, I will introduce a problem of optimizing the utility derived from the consumption and terminal wealth processes for a portfolio of a small, single agent. In a 1987 paper Karatzas, Lehoczky and Shreve showed that under rather mild assumptions they could extend results of Merton's with regards to such a problem. Though they were able to provide explicit information about the optimal consumption and wealth processes, except in the case of deterministic coefficients, they were not able to shed much light on the portfolio process itself. I will conclude the talk by constructing explicitly a portfolio process without the restriction to deterministic coefficients.

The Mode-Switching Model for Stock Price and An Optimal Selling Rule (Feb. 10)
Ruihua Liu
Abstract: The geometric Brownian motion (GBM) model for stock prices is widely used in option pricing and portfolio management. This model has two parameters, the expected rate of return and the volatility, both are assumed to be deterministic constants. The main drawback of this model is its failure to capture the general market movement. A number of modifications with random parameters have been studied.

In recent years interests have focused on the so-called mode-switching model, which incorporates the general market trend into the stock price modeling. In this setting, the parameters depend on the market mode that switches among a number of different states (e.g., bull and bear markets). Mathematically, the mode is modeled by a finite state Markov chain. As a result, we have a hybrid system that switches among a number of GBMs.

One important issue in portfolio management is to decide when to sell a stock. A selling rule is characterized by two numbers: a target price and a stop-loss limit. The stock is sold whenever the price reaches either of the two levels. The objective is to choose these numbers so as to maximize an expected utility function.

In this talk we first introduce the mode-switching model for stock price movement and mention some current research topics related to this model. Then we present our research of optimal selling rule. We formulate this financial problem as a mathematical optimization problem. The optimal policy can be obtained by solving a set of two-point boundary value differential equations. Analytical solutions are obtained for one- and two-state cases. For general case, we develop a stochastic recursive algorithm that can be easily implemented to calculate an approximation of the optimal value. Finally, we mention some topics under consideration for future research.

I**ntroduction to Stochastic Differential Equations and Financial Derivatives** (Feb. 16)
Craig Calcaterra
Abstract: The analysis of stochastic differential equations (SDE's) draws on a wide range of deep ideas from probability and partial differential equations, physics and economics. I will introduce Ito differentiation and Ito integration of random variables with respect to Brownian motion. We then will derive and solve the Black-Scholes partial differential equation for European financial derivatives.

Most of the talk will focus on heuristics and will be accessible to anyone who understands what a random variable is. However the lecture follows the modern mathematical development of the subject, and it will be easy to justify each step with comments on their foundation in measure theory.

Fortune's Algorithm for Constructing Voronoi Diagrams (Mar. 11)
Colleen Livingston
Abstract: Voronoi diagrams partition the plane into zones of nearest proximity to fixed sites in a region. This talk gives an introduction to Voronoi diagrams and two brute force algorithms for constructing these diagrams. Fortune's algorithm is a "sliding line" algorithm. A simple example of sliding lines will be shown and the geometry of Fortune's algorithm explained.

Is global warming injecting randomness into the climate system? (Mar. 25)
Anastasios Tsonis
Abstract: In this talk an investigation into some of the dynamical properties of the climate system is presented. The results are based on modern data analysis techniques that probe the nonlinear character of the system. Two different approaches are considered. One approach finds predictability as a function of time of a very strong signal of the system. It is found that predictability in time is highly correlated to global temperature. More specifically, as the global temperature increases predictability decreases. The other approach studies the collective behavior of the climate system using network dynamics and concludes that this behavior is consistent with a network of increasing randomness. Thus, both approaches suggest that the collective behavior of the climate system may becoming more random. This may present yet, another potential pitfall of global warming.

Pebbling on directed graphs (Mar. 25)

Gayatri Gunda

Abstract: Pebbling on graphs is a relatively new area in mathematics. Its origins are tied to the solution of a number theory problem by Fan Chung in 1989. Most of the work that has been done so far in pebbling has concentrated on undirected graphs. This work initiates the study of pebbling on directed graphs and begins the process of building an analogous theory.

Consider a finite connected graph G whose vertices are labeled with non-negative integers representing the number of pebbles on each vertex. A pebbling move on a graph G is defined as the removal of two pebbles from one vertex and the addition of one pebble to an adjacent vertex. The pebbling number f(G) of a connected graph is the least number of pebbles such that any distribution of f(G) pebbles on G allows one pebble to be moved to any specified but arbitrary vertex. We consider pebbling on directed graphs and study what configurations of directed graphs allow for pebbling to be meaningful.

We also obtain the pebbling numbers of certain orientations of directed wheel graphs with odd order and directed complete graphs with odd order. G is said to be demonic if $f(G) = n$ where n is the order of G. We demonstrate the existence of demonic directed graphs. We establish the surprising result that the sharp upper bound and sharp lower bound of the pebbling numbers of directed graphs on n vertices is the same as that of the undirected graphs: $n < f(G) < 2n-1$.

This work partially fulfills the requirements for a Berry Scholars thesis. We gratefully acknowledge the generous support of the Berry Scholars Program. A paper based on this work has been accepted for publication by the Proceedings of the Undergraduate Mathematics Day conference held at UD in November 2003.

Do I Have a Thumb Twin?

Christopher Ryan

Abstract: Popular opinion indicates that every person who has ever lived has a unique thumbprint. However, without applying mathematical rigor to the problem, this hypothesis is little more than a myth. The focus of this paper is to develop a model to test the hypothesis and to calculate the probability that thumbprints are unique. Furthermore, the model analyzes special limiting cases of the problem. Lastly, the odds of misidentification by way of thumbprint analysis are compared to the odds of misidentification through DNA evidence. The results indicate that both thumbprint and DNA analysis supply individuals with unique identifiers, but a misidentification by DNA is thousands of orders of magnitude less likely.