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Sequential Decreasing Strong Size Properties

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Hyperspaces

- $2^X = \{A \subset X : A \text{ is compact and nonempty}\}$
- $C_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ components}\}$
- $C(X) = \{A \in 2^X : A \text{ is connected}\}$.

Definition

A *Whitney map* for $C_n(X)$ is a map $\omega : C_n(X) \to \mathbb{R}$ such that:

- $\omega(\{x\}) = 0$ for all $x \in X$ and,
- $\omega(A) < \omega(B)$ whenever $A \subset B$.

A Whitney level is a set of the form $\omega^{-1}(t)$ for some $t \in [0, \omega(X)]$

Whitney levels for $C_m(X)$ are not necessarily connected!

Definition (Hosokawa(2011))

A *strong size map* for $C_n(X)$ is a map $\mu : C_n(X) \to \mathbb{R}$ such that:

- $\mu(A) = 0$ whenever $A \in F_n(X)$ and,
- $\mu(A) < \mu(B)$ whenever $A \subset B$ and $B \notin F_n(X)$.

Theorem (Hosokawa(2011))

Strong size maps always exist for continua

A *strong size level* is a set of the form $\mu^{-1}(t)$ for some $t \in [0, \mu(X)]$

Theorem (Hosokawa(2011))

Strong size levels for $C_n(X)$ are continua

Definition

A topological property $\mathcal{P}$ is a *sequential decreasing strong size property(sequential decreasing Whitney property)* provided that if μ is a strong size map(Whitney map) for $C_n(X)$, $\{t_n\}_{n\in\mathbb{N}}$ is a sequence in the interval $(t, 1)$ such that $t_n \to t$ and each fiber $\mu^{-1}(t_n)$ has the property $\mathcal{P}$, then $\mu^{-1}(t)$ has the property $\mathcal{P}$.

Theorem

Being Kelley continuum, unicoherence, indecomposability, local connectedness and continuum chainability are sequential decreasing strong size properties.

Definition

A topological property $\mathcal{P}$ is a *increasing strong size property* provided that if μ is a strong size map for $C_n(X)$ and $t_0 \in [0,1)$ is such that $\mu^{-1}(t_0)$ has property $\mathcal{P}$, then $\mu^{-1}(t)$ has property $\mathcal{P}$ for each $t \in (t_0, 1)$.

Theorem

Being an uniformly continuum-chainable, continuum chainability, arcwise connectedness and locally connectedness are increasing strong size properties.

A *strong size block* is a subset of the form $\mu^{-1}([s, r])$ for a strong size map μ and $0 \leq s \leq r \leq 1$.

Theorem (Hosokawa(2011))

Strong size blocks for $C_n(X)$ are continua.

Theorem

If $t \in [0,1]$, then $\mu^{-1}([t,1])$ is unicoherent. In particular $C_n(X)$ is unicoherent.

Theorem

If $\mu^{-1}(t_0)$ is an arcwise connected continuum and $t_0 \leq t$, then the block $\mu^{-1}([t_0,t])$ is an arcwise connected continuum.

Theorem

If $\mu^{-1}(t_0)$ is a local connected continuum and $t_0 \leq t$, then the block $\mu^{-1}([t_0,t])$ is a local connected continuum.

Theorem

The property of being a irreducible continuum is not a stronger size property for $n \geq 3$.