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On the Lindelöf Σ-Property and Some Related Conclusions

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Outline

Lindelof Σ-spaces

In 1969, Nagami published his paper about Σ-spaces. Nowadays the class of Σ-spaces with the Lindelöf property is important in Topology, Functional Analysis, Topological Algebra and Descriptive Set Theory.

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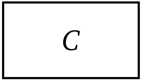

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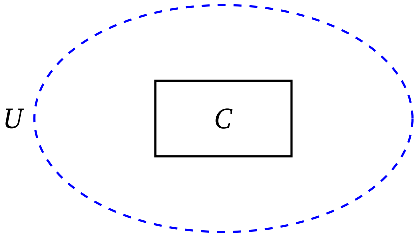

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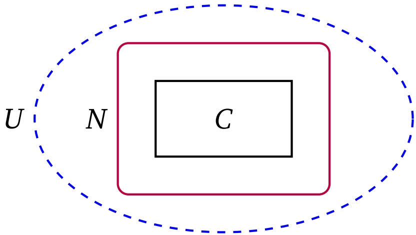

Outline

Σ_s-products

The concept of Σ_s-product was introduced by G. A. Sokolov in 1984, who proved that a compact space X is Gul'ko (i.e., $C_p(X)$ is a Lindelöf Σ-space) if and only if X embeds into a Σ_s-product of real lines.

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Definition. Let $X = \prod_{t \in T} X_t$, fix a point $a \in X$ and a countable family s of subsets of T. Given $x \in X$ let

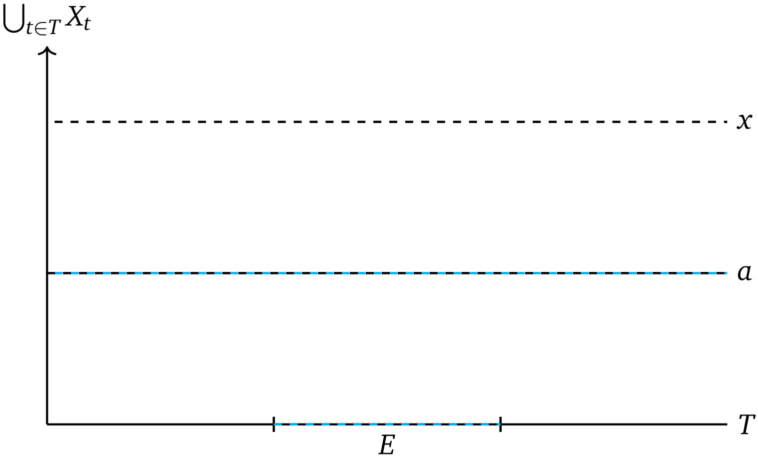
$$s_x = \{E \in s : |supp(x, a, E)| < \omega\}.$$

The Σ_s-*product* of the family $\{X_t\}_{t \in T}$ centered at a with respect to the sequence s is the space:

$$\Sigma_s(X, a) = \{x \in X : T = \bigcup s_x\}.$$

Σ_s-products

$$supp(x, a, E) = \{t \in E : x(t) \neq a(t)\}$$



The Lindelöf Σ-property in Σ_s-products

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Theorem. *Let $X = \prod_{t \in T} X_t$ be a product. Then, every Σ_s-product in X is Lindelöf Σ iff each σ-product in X is Lindelöf Σ.*

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Corollary. *If X is Lindelöf Σ, then every Σ_s-product in X^T also is a Lindelöf Σ-space.*

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Theorem. *If $X = \prod_{t \in T} X_t$ is a product of $\mathcal{K}$-analytic spaces, then each Σ_s-product in X is Lindelöf Σ.*

Outline

1. The Lindelöf Σ-property in Σ_s-products
2. The Lindelöf Σ-property in spaces $C_p(X)$
3. The Lindelöf property in spaces $C_p(X)$
4. Lindelöf subspaces of Σ-products
5. The Nagami number
6. Some related results

As we have mentioned, a compact space X satisfy that $C_p(X)$ is a Lindelöf Σ-space if and only if X embeds into a Σ_s-product of real lines. It would be interesting to determine when $C_p(\Sigma_s\mathbb{R}^T)$ is a Lindelöf Σ-space.

$$C_p(\Sigma_s\mathbb{R}^T) \xrightarrow{\pi_X} C_p(X)$$

$$\Sigma_s\mathbb{R}^T \hookleftarrow X$$

The Lindelöf Σ-property in $C_p(X)$

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Theorem. *Let X be a Lindelöf Σ-subspace of $\Sigma_s\mathbb{R}^T$. Then $C_p(X)$ is a Lindelöf Σ-space.*

A sketch of the proof

Lemma. *The space* $\Sigma_s 2^T$ *can be embedded in* $C_p(L, 2)$ *for some simple Lindelöf* Σ*-space* L*.*

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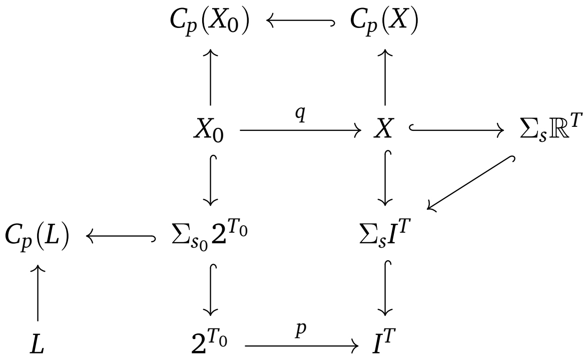

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Theorem. (Tkachuk, 2007) *If $C_p(X)$ is a Lindelöf Σ-space, then X condenses into a Σ_s-product of real lines.*

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However, Lindelöf Σ-spaces X for which $C_p(X)$ is Lindelöf Σ can not be characterized using embeddings in Σ_s-products of real lines.

Example. *There is a Lindelöf Σ-space which can not be embedded in any Σ-product of real lines but such that $C_p(X)$ is Lindelöf Σ.*

We do not know what happens if we consider condensations instead of embedings.

Outline

The Lindelöf property in $C_p(X)$

Considering the previous results, it is interesting to determine if a similar result holds for Lindelöf (Lindelöf Σ) subspaces of Σ-products of real lines.

$$\begin{array}{ccc}
C_p(X) \text{ Lindelöf} & & C_p(X) \text{ Lindelöf } \Sigma \\
\uparrow & & \updownarrow \\
X \text{ Corson compact} & & X \text{ Gul'ko compact} \\
\updownarrow & & \updownarrow \\
X \subset \Sigma\mathbb{R}^T & & X \subset \Sigma_s\mathbb{R}^T
\end{array}$$

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Theorem. *If X is a Lindelöf Σ-subspace of $\Sigma\mathbb{R}^T$, then $C_p(X)$ is Lindelöf.*

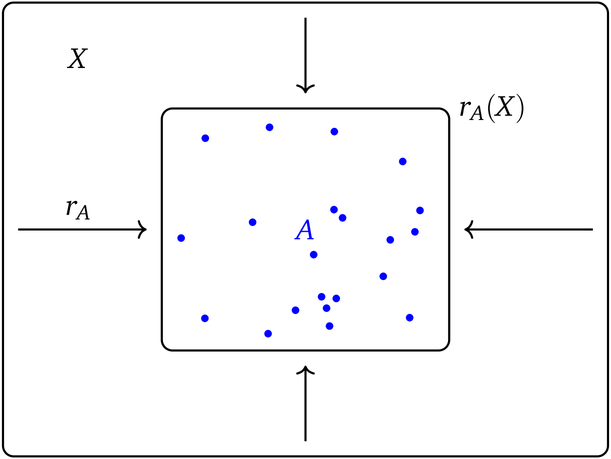
Monotonically retractable spaces

Monotonically retractable spaces where introduced by Rojas in order to study the Lindelöf and the D-property in function spaces.

Definition. A space X is *monotonically retractable* if for each countable set $A \subset X$ we can assign a continuous retraction $r_A : X \to X$ and a countable family $\mathcal{N}(A)$ of subsets of X in such a way that:

1. $A \subset r_A(X)$.
2. $\mathcal{N}_A$ is a network for the weak topology generated by r_A.
3. $A \subset B$ imply $\mathcal{N}(A) \subset \mathcal{N}(B)$;
4. if $\{A_n\}_{n\in\omega}$ is increasing, then
$$\mathcal{N}(\bigcup_{n<\omega} A_n) = \bigcup_{n<\omega} \mathcal{N}(A_n).$$

Monotonically retractable spaces



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Theorem. (Rojas-Hernández, 2014) *If X is monotonically retractable, then $C_p(X)$ is Lindelöf D.*

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Example. *The space $\Sigma\mathbb{R}^T$ is always monotonically retractable.*

$$C_p(\Sigma\mathbb{R}^T) \xrightarrow{\pi_X} C_p(X)$$

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Theorem. *If X is monotonically retractable and Y is a Lindelöf Σ-subspace of X, then Y is monotonically retractable.*

In particular, if X is a Lindelöf Σ-subspace of $\Sigma\mathbb{R}^T$, then each $C_{p,n}(X)$ is Lindelöf.

The idea of the proof

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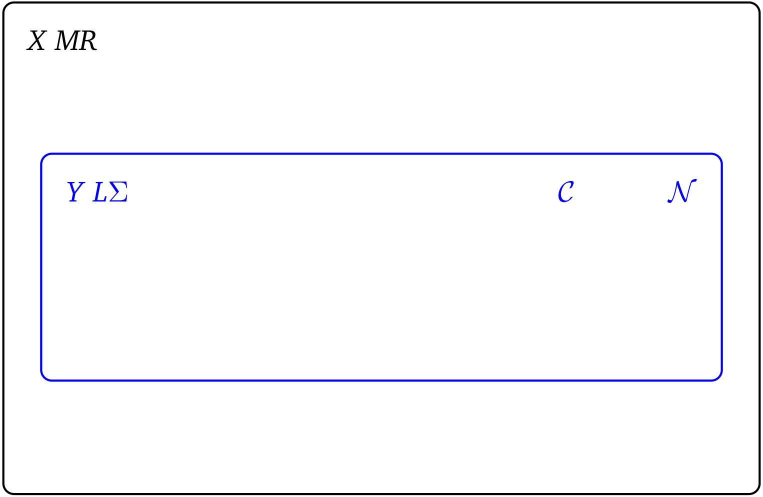

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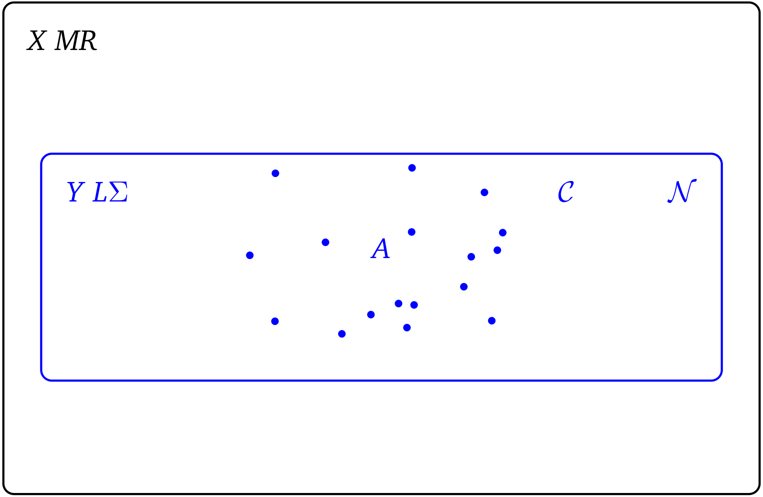

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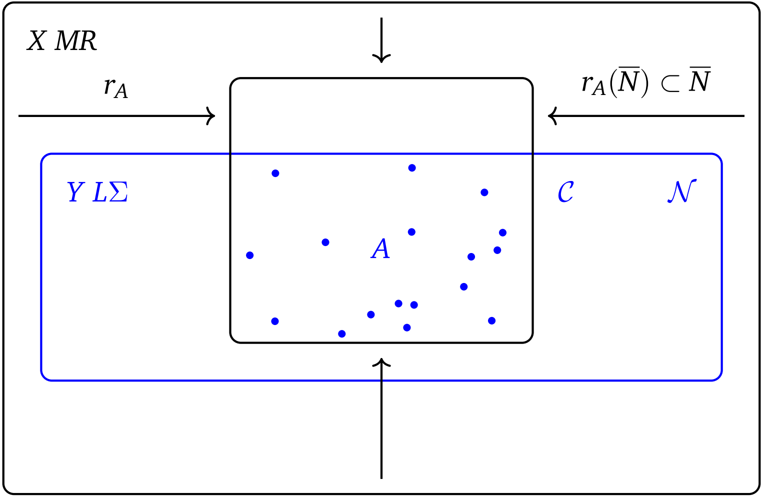

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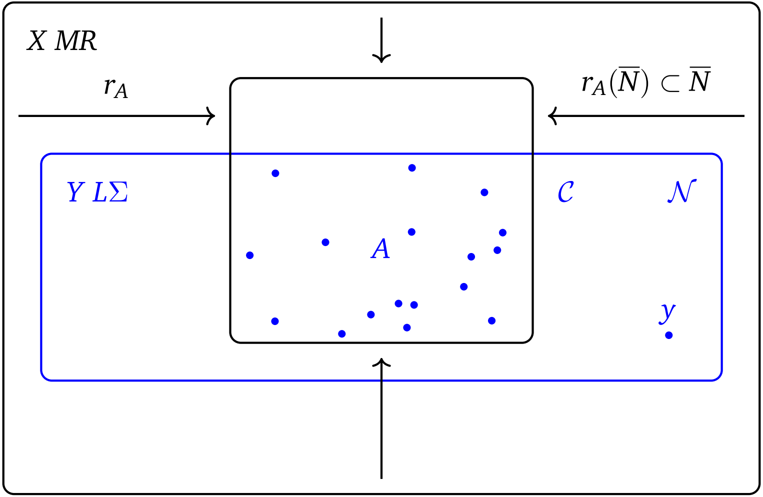

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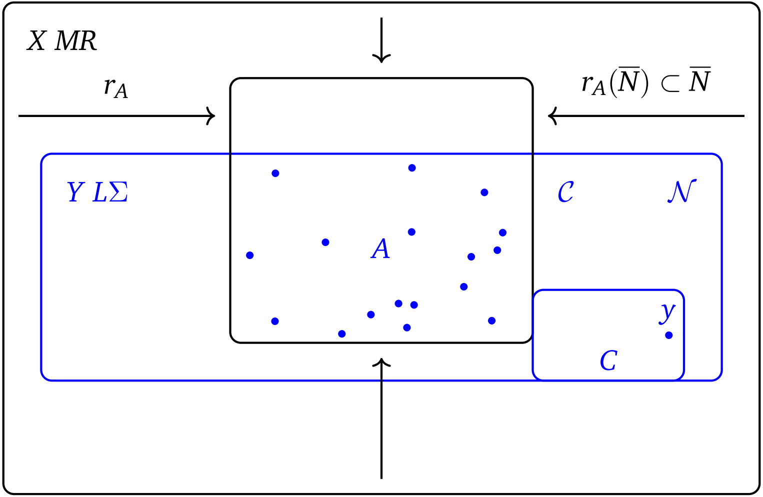

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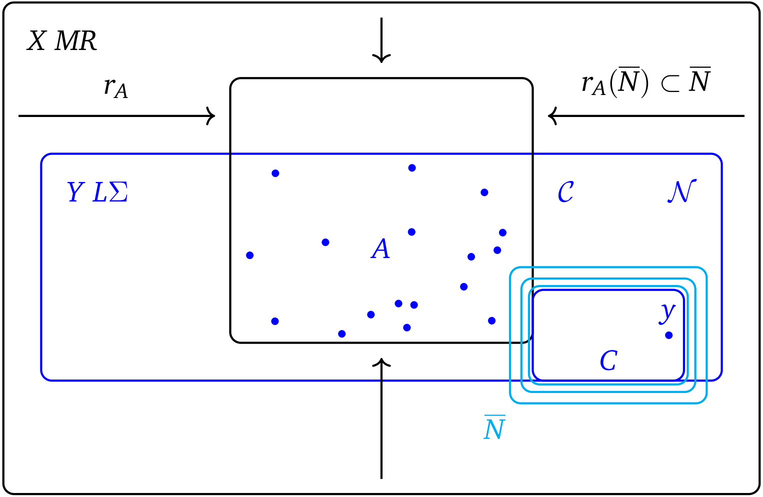

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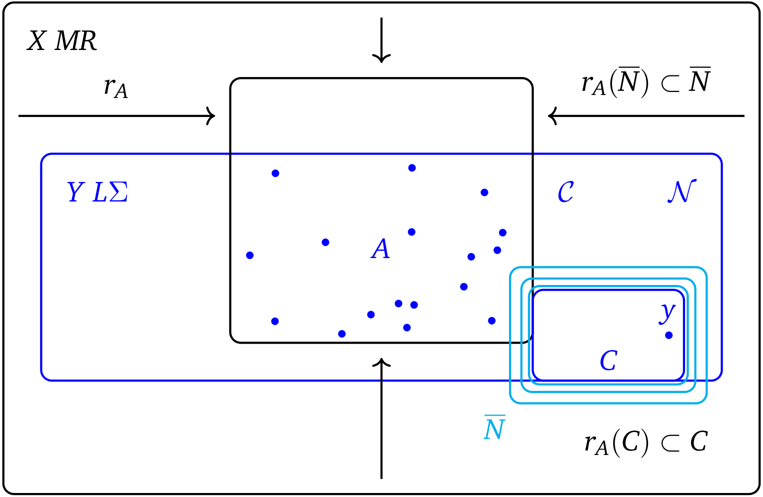

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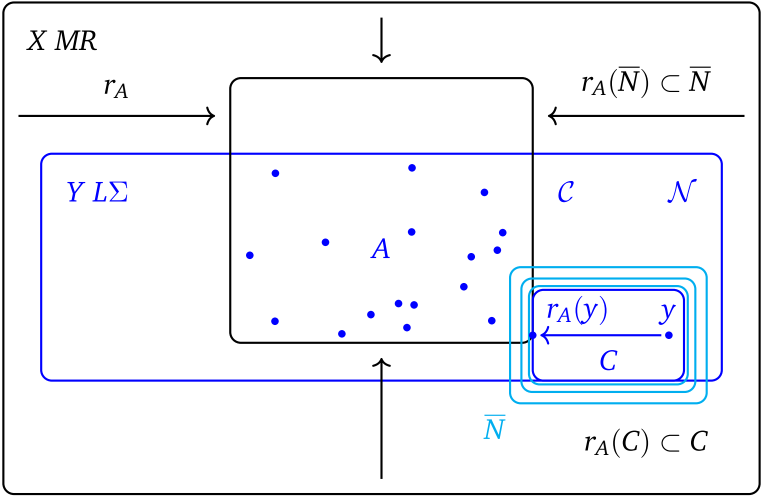

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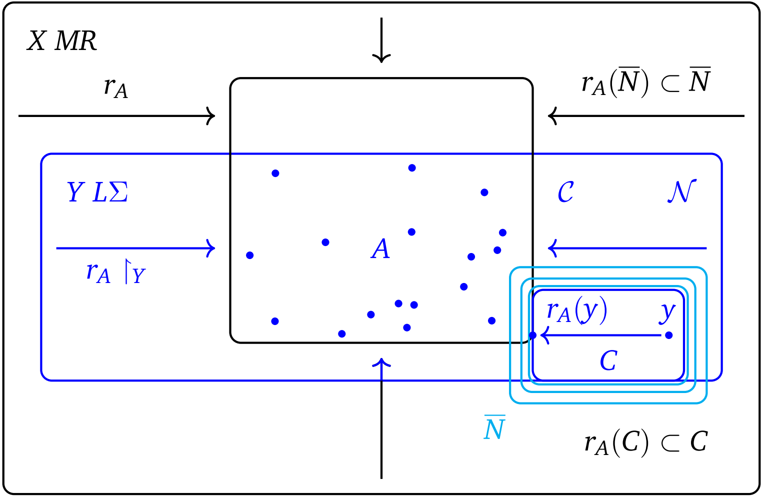

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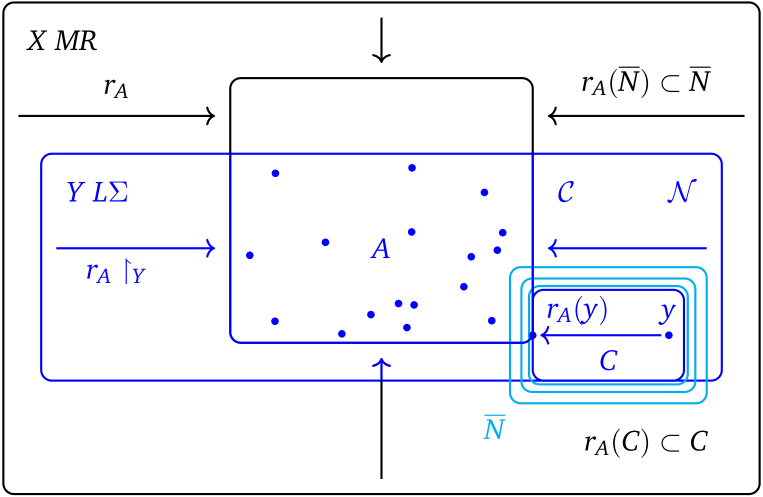

Outline

Lindelöf subspaces of Σ-products

Example. *There exists a Lindelöf non Lindelöf Σ subspace X of a Σ-product of real lines such that $C_p(X)$ is Lindelöf.*

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Let K be a countable tight Sokolov non-Gul'ko compact space of cardinality ω_1 with an unique complete accumulation point.

$$\begin{array}{ccccc} & & C_p(X) & & \\ & & \uparrow & & \\ C_p(K) \sim X \times \mathbb{R} & \hookleftarrow & X & \hookrightarrow & \Sigma\mathbb{R}^T \\ \uparrow & & & & \\ K = T \cup \{p\} & & & & \end{array}$$

$$X = \{f \in C_p(K) : f(p) = 0\}$$

Lindelöf subspaces of Σ-products

Theorem. (Tkachuk 2017) *There is a Lindelöf P-space X such that* $C_p(X)$ *has uncountable extent. In particular,* $C_p(X)$ *is not Lindelöf.*

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For a compact X we have the following:

$$\begin{array}{ccc} \Sigma\mathbb{R}^T & \Sigma_s\mathbb{R}^T & \Sigma_*\mathbb{R}^T \\ \big\uparrow & \big\uparrow & \big\uparrow \\ X \text{ Corson} & X \text{ Gul'ko} & X \text{ Eberlein} \end{array}$$

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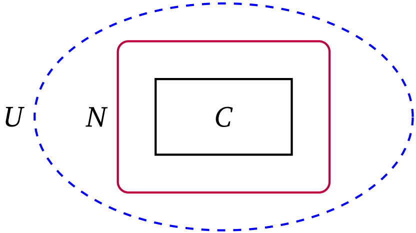
X Corson	X Gul'ko	X Eberlein

Example. *If there exists a Souslin line, then we can find a Corson compact space with a Lindelöf non Lindelöf Σ-subspace.*

Outline

The Nagami number

Definition. The *Nagami number* of a space X is the minimal cardinality of a network $\mathcal{N}$ with respect to a compact cover $\mathcal{C}$ of X.



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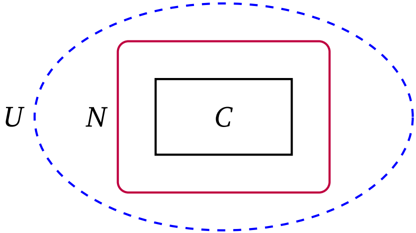

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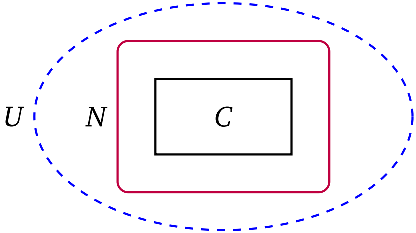

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$$l(X) \leq Nag(X) \leq nw(X).$$

A bound using the Nagami number

Looking for bounds of the weight of Lindelöf Σ-subspaces of separable Haussdorf spaces, Tkachenko obtained the following:

Theorem. (Tkachenko, 2015) *Given a Tychonoff space X we have:*

$$w(X) \leq | C(X) | \leq nw(X)^{Nag(X)}.$$

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Corollary

- *If X is a Lindelöf Σ-space, then $w(X) \leq nw(X)^{\omega}$.*
- *If X is an $L\Sigma(\leq \mathfrak{c})$-space, then $w(X) \leq \mathfrak{c}$.*

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The proof of this inequality involves strongly the use of C_p-theory tools. He asked for a direct proof of this fact.

Theorem. *The bound $w(X) \leq nw(X)^{Nag(X)}$ holds for each regular space X.*

Outline

t_n-equivalences

The iterated function spaces $C_{p,n}(X)$ over a space X are recursively defined as follows:

$$C_{p,0}(X) = X \text{ and } C_{p,n+1}(X) = C_p(C_{p,n}(X)).$$

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Theorem. (Okunev, Tkachuk) *A Gul'ko compact space is Gulko if and only if $C_{p,n+1}(X)$ is Lindelöf Σ for some $n \in \omega$.*

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Theorem. *A compact space is Corson if and only if $C_{p,n}(X)$ has a strong r-skeleton for some $n \in \omega$.*

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Theorem. *A compact space is Corson if and only if $C_{p,n}(X)$ has a strong r-skeleton for some $n \in \omega$.*

Theorem. *A compact space is Eberlein if and only if both $C_{p,n}(X)$ and $C_{p,n+1}(X)$ have a dense σ-compact subspace for some $n \in \omega$.*

Collins-Roscoe spaces

An equivalence of the following notion was introduced by Collins and Roscoe to study conditions for metrizability.

Definition. A space X has the *Collins-Roscoe property* if we can assign, to each $x \in X$, a countable family $\mathcal{N}(x)$ of subsets of X satisfying

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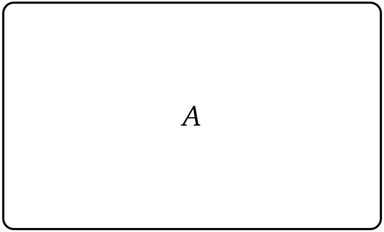

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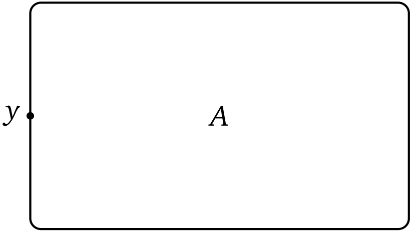

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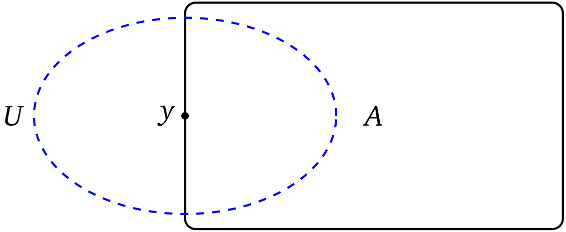

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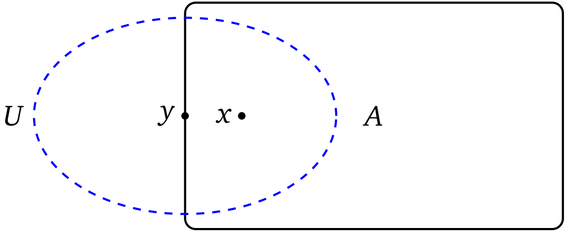

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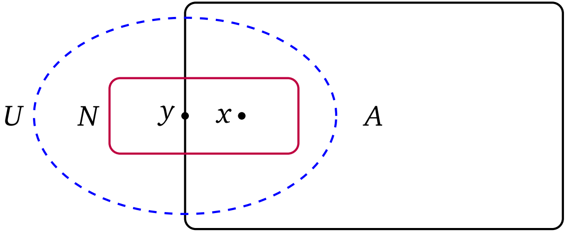

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Questions

- Must every Σ_s-product of Lindelöf Σ-spaces be Lindelöf Σ?
- Find a characterization of Lindelöf Σ-spaces (or σ-compact spaces) for which $C_p(X)$ is Lindelöf Σ.
- Is the space $C_p(X)$ Lindelöf for each Lindelöf subspace of a Corson (Gul'ko or Eberlein) compact space?
- Holds the bound $w(X) \leq nw(X)^{Nag(X)}$ for Hausdorff spaces? (Tkachenko)
- Find an internal property in $C_{p,n}(X)$ which caracterizes the Eberlein property in compact spaces.
- Is it true that $C_p(X)$ is a Collins-Roscoe space whenever X is a Corson compact space? (Tkachuk)

Thank You!