4-17-2013

The Kou Jump-Diffusion Model for Option Pricing

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Kou’s Model for Option Pricing
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Why Kou’s Model?
The Black-Scholes model has been a useful tool for option pricing in the financial market. However, there are two phenomena – the leptokurtic feature and the implied volatility curve – which it fails to capture. We looked at Kou’s model which accounts for these phenomena, and it leads to an analytical solution for many option pricing problems.

Kou’s Formula:
Below is the formula derived in Kou’s article. We can see that he uses a double summation that calls several other functions.

\[
Y(x; \sigma, \lambda, p, \eta_1, \eta_2; a, T) = \frac{e^{(a \eta_2 / \sigma^2)T/2}}{\sigma \sqrt{2\pi} T} \sum_{k=1}^{a} \sum_{l=1}^{a} \rho_{kl}(\sigma \eta_1 \sqrt{T})^k \left( a - \kappa T; -\eta_1, -\frac{1}{\sigma \sqrt{T}}, -\sigma \eta_1 \sqrt{T} \right) + e^{(a \eta_2 / \sigma^2)T/2} \sum_{k=1}^{a} \sum_{l=1}^{a} Q_{kl}(\sigma \eta_2 \sqrt{T})^k \left( a - \kappa T; -\eta_2, -\frac{1}{\sigma \sqrt{T}}, -\sigma \eta_2 \sqrt{T} \right) + \pi \sigma \left( a - \kappa T \right) \sqrt{T} \pi \sigma \left( a - \kappa T \right) \sqrt{T}
\]

Kou used the Upsilon equation to find the value of the option.

\[
V(0) = S_0 Y \left( r - \beta + \frac{1}{2} \sigma^2, \alpha, \lambda, p, \eta_1, \eta_2; \ln \left( \frac{K}{S_0} \right), T \right) - Ke^{-rT} Y \left( r - \beta + \frac{1}{2} \sigma^2, \alpha, \lambda, p, \eta_1, \eta_2; \ln \left( \frac{K}{S_0} \right), T \right)
\]

The \( Hh \) function can be viewed as a cumulative normal distribution function where the left tail has a polynomial growth rate and the right tail has an exponential decay.

Below are the \( Hh \) and \( l_n \) functions which are called in the Upsilon function above.

\[
Hh_n(x) = \int_x^\infty Hh_{n-1}(y)dy = \frac{1}{n} \int_x^\infty (t-x)^n e^{-\frac{t^2}{2}} dt
\]

\[
l_n(c; \alpha, \beta, \delta) = \int_c^\infty e^{\alpha x} Hh_n(\beta x - \delta) dx
\]

Density Function for Kou Model:
Below is the density function used in Kou’s model.

\[
f(x) = \begin{cases} 
\frac{\rho_{11} e^{-\eta_1 x^2}}{1 - p} & \text{if } x \leq 1 \\
\frac{\rho_{22} e^{-\eta_2 x^2}}{1 - p} & \text{if } x > 1 
\end{cases}
\]

Kou’s density function (blue) is plotted against the normal curve (red). It is clear to see that the Kou’s density function accounts for discontinuity and the fat tails.

MatLab and Results:
Below is a diagram that shows the MATLAB implementation and how the functions were called.

This table shows the values of European put options with changing stock prices.

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<th>Stock Price</th>
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