Solving Crime Using Mathematics

Michael Jacob
Advisor: Muhammad Usman, Ph.D.

Abstract: Mathematics is used in almost every area of life. With the development of modern computers, mathematical modelling and numerical simulation is new synergy in scientific discovery. In this work nonlinear equations are solved in order to determine the time of death to solve a crime. The equations are solved with few methods and we compare the accuracy of methods.

Introduction
Forensics is among the countless applications of mathematics. Mathematical modelling is used in almost every area of science, engineering and social sciences as well. One of the interesting application of mathematical modelling and simulation is forensic sciences. In this work we study a mathematical model for determining the time of death. Here is an interesting situation [1]:

Commissioner Gordon had been found dead in his office. At 8:00 pm, the county corner determined the core temperature of the corpse to be 90°F. One hour later, the core temperature had dropped to 85°F. Captain Furillo believed that the infamous Doc B had killed the commissioner. Doc B, however, claimed to have an alibi. Lois Lane was interviewing him at the Daily Plant Building, just across the street from the commissioner’s office. The receptionist at the Daily Plant Building checked Doc B into the building at 6:35 pm, and the interview taps confirmed that Doc B was occupied from 6:40 pm until 7:15 pm.

To support or disprove his alibi, one must determine the time of death.
This can be done by modeling Newton’s Law of Cooling.
Taking t = 0 to correspond to 8:00 pm, one can find the equations
\[ 73 - \frac{1}{k} + \left(18 + \frac{1}{k}\right) e^{-kt} = 85 \]
\[ 72 + t_d - \frac{1}{k} + \left(18 + \frac{1}{k}\right) e^{-kt_d} = 98.6 \]

Where k is the constant of proportionality
These equations will be solved using 4 different methods.

Mathematical Methods Used

• Method 1 – Newton’s Method
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

• Method 2
\[ x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'(x_n)^2 - f(x_n)f''(x_n)} \]

• Method 3
\[ y_n = x_n - \frac{f(x_n)}{f'(x_n)} \]
\[ x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'(x_n)^2 - f(x_n)f''(y_n)} \]

• Method 4
\[ y_n = x_n - \frac{f(x_n)}{f'(x_n)} \]
\[ x_{n+1} = y_n - \frac{2f(y_n)f'(y_n)}{2f'(y_n)^2 - f(y_n)f''(y_n)} \]

Analysis

• The first equation was solved for k using Newton’s method, then second equation for t_d.
• k was found to be 0.33711438097
• Methods 1-4 were applied until the approximate error for each fell into a certain range
• Approximate error is calculated using the equation
\[ \text{Error} = \frac{x_{n+1} - x_n}{x_n} \]

• The table below shows that the 4 methods [2] began to converge on a single value for

<table>
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<th>Approximate Error</th>
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<th>Method 2</th>
<th>Method 3</th>
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</table>

Conclusion

• The value -1.130938140002211 means that the time of death was roughly 1 hour and 8 minutes before 8:00 pm
• The suspect’s alibi is upheld

References

[1]. A friendly introduction to numerical analysis by Brian Bradie