A Numerical Solution of a Model of Diabetes

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A Numerical Solution of a Model of Diabetes
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Numerical Methods Used for Simulations

The Glucose Insulin Interaction Model was simulated using three methods: Runge-Kutta methods of orders two and four, and the Adams-Bashforth Method.

Runge-Kutta Methods

Runge-Kutta Order 2:

\[ x(t+h) = x(t) + \frac{h}{2} (k_1 + k_2) \]

\[ k_1 = f(t, x) \]

\[ k_2 = f(t + h, x + k_1) \]

Runge-Kutta Order 4:

\[ x(t+h) = x(t) + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4) \]

\[ k_1 = f(t, x) \]

\[ k_2 = f(t + \frac{h}{2}, x + \frac{1}{2}k_1) \]

\[ k_3 = f(t + \frac{h}{2}, x + \frac{1}{2}k_2) \]

\[ k_4 = f(t + h, x + k_3) \]

Adams-Bashforth Method:

\[ x_{n+1} = x_n + \frac{h}{2} (3f(x_n, t_n) - f(x_{n-1}, t_{n-1})) \]

Simulations

These plots show that the steady, slow increase in insulin as a result of increased glucose concentration serves to lessen the rate at which the glucose concentration increases. These simulations are extremely close to the graphical solution modelled by Hussain and Zadeng.9 As indicated by Hussain and Zadeng, when the value of \( a_1 \) is larger, the peak glucose concentration is smaller which shows that insulin is important to the regulation of glucose concentration in the human body.9 This information is important to know so that doctors and patients alike can have a better understanding of how to regulate glucose and insulin concentrations.

Models similar to this simulated model by Hussain and Zadeng can be used for a variety of things. For example, determining how sodium bicarbonate will interact with pH levels of chlorinated bodies of water, such as pools. Sodium bicarbonate is used to increase alkalinity which in turn acts as a buffer to resist changes in pH in bodies of water; however the addition of sodium bicarbonate to a body of water will increase the pH. A differential equation modeling the concentration of sodium bicarbonate and an equation modeling the concentration of hydrogen ions can be used to better predict how adding bicarbonate to increase water alkalinity will affect the pH over a period of time. This could be useful information to know because city health departments require the pH of pools to be within a specific range (typically between 7.2 and 7.6), and adding sodium bicarbonate can greatly affect the pH staying within the designated range.

Discussion and Future Work

These plots show that the steady, slow increase in insulin as a result of increased glucose concentration serves to lessen the rate at which the glucose concentration increases. These simulations are extremely close to the graphical solution modelled by Hussain and Zadeng.9 As indicated by Hussain and Zadeng, when the value of \( a_1 \) is larger, the peak glucose concentration is smaller which shows that insulin is important to the regulation of glucose concentration in the human body.9 This information is important to know so that doctors and patients alike can have a better understanding of how to regulate glucose and insulin concentrations.

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References and Acknowledgments


Nullcline Model

The use of nullclines is important to the qualitative analysis of the model that was previously simulated. This is because nullclines allow a conclusion to be made regardless of the solution of the system of differential equations.10 Through the creation of a phase plane, the nullclines are sets of points where \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \) are equal to zero. In a phase plane, the direction of motion will converge to these points where \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \) are equal to each other.10 In Fig. 4 below, a phase plane of the model simulated using MATLAB code from John C. Poulton at MathWorks6 is shown.

As can be seen in Fig. 4, the lines converge to a point somewhere to the right of what can be displayed in this nullcline model. The fact that these lines converge indicates that the model used by Hussain and Zadeng is asymptotically stable, which is something they proved numerically in their discussion of the model.11 Asymptotic stability is important to recognize because that means that no matter what the initial conditions may be, the model will always approach the same equilibrium point.10 This means that this particular model can be used for any individual afflicted with diabetes.

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