Parameter Identification in Structured Discrete-Time Uncertainties without Persistency of Excitation
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**Background**

- **System Identification:** Function approximation

  ![Uncertain Signal](input -> uncertain f -> output)

- **Sys-ID usage:** machine learning, adaptive control, . . .
- **Present study:**
  - Discrete-time (DT) structured uncertainties
  - $f(x(k)) = \theta^T \phi(x(k))$
  
  $$f(x(k)) = \frac{1}{4} \cdot 10 \exp \left( \frac{-|x(k) - x_0|^2}{4} \right) = \frac{1}{b(x(k))} \left[ \frac{1}{\theta^T} \phi(x(k)) \right]$$

- Example:
  - $f_i(x(k)) = \frac{10}{4} \exp \left( \frac{-|x(k) - x_0|^2}{4} \right) = \frac{1}{b(x(k))} \left[ \frac{1}{\theta^T} \phi(x(k)) \right]$

- Approximator
  - $\theta_i(k) \in \mathbb{R}^n$: estimate of $\theta_i$ updated via adaptation law
  - Parameter error $\hat{\theta}(k) \neq \theta$ (not computable)

- Compute estimation error
  - $q(k) = \frac{1}{\theta^T} \phi(x(k))$
  - $\hat{\theta}(k) \leftarrow \hat{\theta}(k) + \eta q(k)$

**Parameter Identification (PI) Problem**

Drive $\hat{\theta}(k) \to [0]^n$ or $\hat{\theta}(k) \to \theta$, causing $q(k) \to 0$, as $k \to \infty$

**Motivation**

- PI, i.e., $\hat{\theta}(k) \to 0$, leads to improved estimation performance
- Literature: traditional approximation methods guarantee PI provided persistency of excitation (very restrictive)
- Present study:
  - Develop an adaptive estimator with PI guarantees
  - Relax persistency of excitation requirement

**Normalized Gradient (NG) Descent**

- NG: traditional approach to approximation
- NG adaptation law: given an initial $\hat{\theta}(k)$,
  $$\hat{\theta}(k+1) = \hat{\theta}(k) - \eta \frac{\phi(x(k)) q(k)}{m^T(k)}$$

- $\eta > 0$: step size or learning rate or gain
- $m(k)$: normalization signal ensuring $\psi(x(k)) = \frac{\phi(x(k))}{m(k)}$
- Lyapunov stability analysis: we can show that $\hat{\theta}(k)$ remains bounded for all $k$ if $0 < \eta < \pi_{NG}$
- PI, i.e., $\hat{\theta}(k) \to 0$, only if $\psi(x(k))$ is persistently exciting

**Concurrent Learning (CL) Preliminaries**

- CL: first introduced in continuous-time framework
- Use of memory:
  - Record past data for $k_0 < \tau_j < k$, with $j = 1, 2, \ldots, c_x$
  - $Z \in \mathbb{R}^{c_x \times n}$: history stack of $\psi(x(\tau_j))$ vectors
  - $\theta \in \mathbb{R}^n$: vector of $\psi(x(k))$ values
  - $\bar{Z} \in \mathbb{R}^{c_x \times n}$: vector of $\psi(x(\tau_j))$ values
- CL condition: $Z$ contains $r_0$ linearly independent $\psi(x(\tau_j))$
- Less restrictive than persistency of excitation

**Gradient-Based CL in DT**

- Gradient-Based CL adaptation law: given an initial $\hat{\theta}(k_0)$,
  $$\hat{\theta}(k+1) = \hat{\theta}(k) - \eta \frac{\phi(x(k)) q(k)}{m^T(k)}$$

- Estimation error based on recorded data:
  $$\hat{\theta}(k) - \hat{\theta}(k) \leftarrow \hat{\theta}(k) - \eta \frac{\phi(x(\tau_j)) q(k)}{m^T(k)}$$

- Lyapunov stability analysis: granted CL condition is met, $\Omega - ZZ^T$ is positive definite and we prove that $\hat{\theta}(k) \to 0$ exponentially (PI) if $0 < \eta < \pi_{CL}$

**Numerical Simulations**

- Here, $f = f_1$ is approximated
- $x$ is varied from $x_L \to -2\pi$ at $k_0$ to $x_H \to +3\pi$ at $k_L$ uniformly
- How good is $f$ if $\hat{\theta}(k)$ is frozen at each $t$ to reconstruct $f$? Consider metric
  $$e(k) = \int_{k_0}^k |f(\hat{\theta}(k)) - f(x)| dx$$

**Future Work**

- How will CL fare with unstructured uncertainties?
- Apply CL adaptation law within a control loop